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Numerical Investigation of Laminar Separated Trailing-Edge Flows

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An efficient time-marching numerical method has been developed for solving problems involving supersonic laminar separated trailing-edge flows using a triple-deck formulation. The accuracy of the numerical method was tested for the case of a flat plate embedded in a supersonic flow at angles of incidence below stall. A range of numerical solutions is presented for angles of incidence beyond stall in the form of streamline and shear contour plots. The trailing-edge shear on the side of the plate where separation occurs reaches an asymptotic value as the angle of incidence is increased; beyond this point numerical instability was encountered.

I. Background

IN a viscous flow over a plane surface, flow separation will occur if a compressive disturbance of sufficient magnitude is encountered. The point of separation is known[1] to be well upstream of the disturbance even though Prandtl's boundary-layer equations, being of parabolic type, permit no upstream propagation of disturbances. If the flow external to the boundary layer is supersonic, there again is no upstream propagation of disturbances. In 1950, Lighthill[2] showed that a theory based on purely inviscid phenomena was unable to account for the extent of upstream influence found in experimental data. This work marked the beginning of efforts to build a theory of viscid/inviscid interaction in supersonic flow.

By assuming that the pressure gradient in the external supersonic flow is large, even though the change in pressure is small, Stewartson[3] was able to show that the inner boundary layer, which develops very close to the surface, will separate before the main boundary layer is significantly altered. Crocco and Lees[4] showed that when the pressure imposed by the outer inviscid flow is coupled to the displacement thickness of the boundary layer, the observed upstream influence could be produced. To provide a mechanism in the boundary layer leading toward separation, Lighthill[5] incorporated a viscous inner layer in his previous work and was able to obtain a self-consistent theory for the viscid/inviscid interaction phenomenon. This work suggested that the length of the interaction region is of the order ε^3, where $\varepsilon = Re^{-1/8} = (X_o U_\infty/\nu_\infty)^{-1/8}$.

In 1969, a rational self-consistent mathematical triple-deck formulation for viscid/inviscid interacting flows was developed by Stewartson and Williams[6] and independently by Messiter[7] and by Neiland.[8] The triple-deck formulation provides the correct asymptotic behavior as the Reynolds number goes to infinity; the result is that the flow near the surface is governed by the incompressible boundary-layer equations, with modified boundary conditions. These boundary conditions incorporate the interaction with the external potential flow.

The triple-deck formulation, or structure, has been employed in numerous viscid/inviscid interaction problems, as indicated in a survey by Stewartson.[9] Some more recent applications include the solution for supersonic flow past a compression corner by Jenson et al.[10,11]; the solution for supersonic flow past the trailing edge of a flat plate at incidence by Daniels[12]; the solution to supersonic flow past both a two- and a three-dimensional hump by Burggraf and Duck[13] and by Smith et al.[14]; and the mathematical formulation of the viscous flow about the trailing edge of a rapidly oscillating flat plate by Brown and Daniels.[15] (See also Maozhao,[16] Werle and Verdon,[17] and Vatsa and Verdon.[18])

This study is concerned with the phenomenon of nonsymmetric, one-sided separation at the trailing edge of a flat plate in a supersonic stream.[19] Studies of the unseparated case have been carried out by Chow and Melnik[20] for incompressible flow and by Daniels[12] for supersonic flow. In both the compressible and incompressible cases it was thought that the numerical results implied a breakdown of the triple-deck structure at an angle of incidence for which separation occurred at the trailing edge (catastrophic stall). However, Mansfield[19] and Elliott and Smith[21] have obtained solutions at larger angles of incidence, where regions of reversed flow extend well forward of the trailing edge. Surprisingly, the reversed-flow region does not extend into the wake, but instead undergoes an abrupt reattachment process at the trailing edge. The results presented here and in Ref. 19 indicate that the shear on the separation side reaches a minimum asymptotic value at the trailing edge, while the results of Elliott and Smith[21] indicate that the shear reaches a minimum value through a sudden plunge and then reaches a positive value before the trailing edge.

II. Governing Equations

The purpose of this work is to numerically investigate laminar trailing-edge separation on a flat plate in a supersonic flow. This task is accomplished by introducing the triple-deck structure and placing the center of the structure at the trailing edge. This structure provides a mechanism to remove the discontinuity of the vertical velocity that arises due to the relaxation of the no-slip condition in the wake.

The triple-deck structure is symmetric about the plate; therefore, only the structure above the plate is indicated in Fig. 1. The streamwise extent of the triple-deck structure is of $O(\varepsilon^3)$ and consists of three distinct vertical regions. In the middle region, called the main deck, the flow is inviscid and

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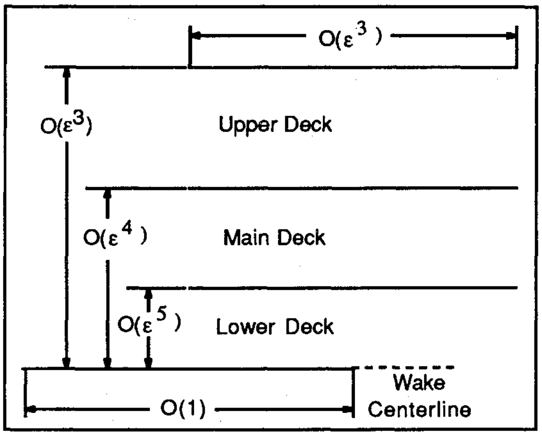


Fig. 1 Triple-deck structure above the plate.

rotational and is of $O(\varepsilon^4)$ in the vertical extent. The function of the main deck is to transmit streamline displacement from the lower deck to the upper deck and pressure perturbations from the upper deck to the lower deck. Using the asymptotic expansions for the main deck, the Navier-Stokes equations, to leading order, reduce to equations in the main deck that correspond to those of simple streamline displacement; the solution of these leading-order equations must match the linear part of the appropriate, fully developed Blasius solution upstream and the Goldstein wake solution downstream.

Located above the main deck is a region with a thickness of $O(\varepsilon^3)$, called the upper deck, where the flow is inviscid and irrotational. The solution for the upper deck must match the inviscid flow solution both upstream and downstream. The governing equations for the upper deck are the Prandtl-Glauert equations, hence the fluid is disturbed by the flow in the interaction region through outward propagation of Mach waves.

The lower deck has a thickness of $O(\varepsilon^5)$ and is located beneath the main deck to satisfy the no-slip condition on the plate. Because of the extreme thinness of the lower deck, the governing equations, to leading order, are the usual unsteady, incompressible boundary-layer equations[14] (momentum and continuity equations) and are given as

$$\frac{\partial u_l}{\partial \tilde{t}} + u_l \frac{\partial u_l}{\partial \tilde{x}} + v_l \frac{\partial u_l}{\partial \tilde{y}} = -\frac{\partial p_l}{\partial \tilde{x}} + \frac{\partial^2 u_l}{\partial \tilde{y}^2} \tag{1a}$$

and

$$\frac{\partial \tilde{u}_l}{\partial \tilde{x}} + \frac{\partial \tilde{v}_l}{\partial \tilde{y}} = 0 \tag{1b}$$

where l and $\sim$ indicate lower-deck scaled variables. The arguments and scalings involved are given elsewhere (Refs. 12, 19, 22, and others). The equations governing the inner boundary conditions are given as

$$u_l \rightarrow |\tilde{y}|,\ p_l^T \rightarrow -\alpha,\ p_l^B \rightarrow \alpha \text{ as } \tilde{x} \rightarrow -\infty \tag{2a}$$

$$p_l \rightarrow 0 \text{ as } \tilde{x} \rightarrow \infty \tag{2b}$$

$$u_l = v_l = 0 \text{ on } |\tilde{y}| = 0 \text{ for } \tilde{x} \leq 0 \tag{2c}$$

and

$$p_l^T(\tilde{x}) = p_l^B(\tilde{x}) = p_l(\tilde{x}) \text{ for } \tilde{x} \geq o \tag{2d}$$

where the superscripts T and B correspond to $y = 0^+$ and $y = 0^-$, respectively. The following unusual outer-boundary condition involving the displacement thickness is obtained from matching the lower deck to the main deck:

$$A^T(\tilde{x}, \tilde{t}) = \lim_{\tilde{y} \to +\infty} (\tilde{u}_l - \tilde{y}) \tag{3a}$$

and

$$A^B(\tilde{x}, \tilde{t}) = \lim_{\tilde{y} \to -\infty} (\tilde{u}_l + \tilde{y}) \tag{3b}$$

where

$$P^T(\tilde{x}, \tilde{t}) = -\frac{\partial A^T}{\partial \tilde{x}} \tag{3c}$$

and

$$P^B(\tilde{x}, \tilde{t}) = -\frac{\partial A^B}{\partial \tilde{x}} \tag{3d}$$

The problem of a flat plate in a supersonic flow is now completely defined.

III. Numerical Method

The numerical method developed in this paper for solving the problem of a flat plate in supersonic flow using the triple-deck structure given in Eqs. (1–3) is an extension to the method successfully applied to the problem of supersonic viscous flow past a compression corner by Jenson et al.[10] A more complete discussion of the method presented here is given in Ref. 19.

The numerical method is simplified through the use of the Prandtl transposition theorem; let

$$t = \tilde{t} \tag{4a}$$

$$x = \tilde{x} \tag{4b}$$

$$y = \tilde{y} + \alpha x - \delta(x, t) \tag{4c}$$

$$u = u_l \tag{4d}$$

and

$$v = v_l - \frac{\partial \delta}{\partial t} - u_l \frac{\partial \delta}{\partial x} + \alpha u_l \tag{4e}$$

where $\delta(x, t)$ is the position of the dividing streamline for $x > 0$, and $\delta(x, t) = 0$ for $x \leq 0$. The dividing streamline is defined as the streamline that divides the fluid particles that pass above the plate from those that pass below the plate; therefore, $v = 0$ at $y = 0$ for all x. The momentum and the continuity equation are invariant under the Prandtl transformation and are given by

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \tag{5}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

The pressure term is formally removed from the problem by solving the following unsteady shear-transport equation:

$$\frac{\partial \tau}{\partial t} + u \frac{\partial \tau}{\partial x} + v \frac{\partial \tau}{\partial y} = \frac{\partial^2 \tau}{\partial y^2} \tag{7}$$

where τ is the nondimensional shear stress defined by

$$\tau = \frac{\partial u}{\partial y} \tag{8}$$

The far-field boundary conditions given in Eq. (2a) reduce to

$$\tau^T \to 1,\ \tau^B \to -1 \text{ as } x \to -\infty \tag{9}$$

On the plate the usual no-slip condition ($u = v = 0$) holds; however, on the dividing streamline only $v = 0$. The velocity on the dividing streamline requires the compatability condition given in Eq. 5 with $v = 0$, which results in

$$\left.\frac{\partial \tau}{\partial y}\right|_{y=0} = \frac{\partial P}{\partial x} + \frac{\partial u_o}{\partial t} + u_o \frac{\partial u_o}{\partial x} \tag{10}$$

where u_o is the velocity on $y = 0$. For $x < 0$ the compatibility condition reduces to

$$\left.\frac{\partial \tau}{\partial y}\right|_{y=0} = \frac{\partial P}{\partial x} \tag{11}$$

The pressure must be evaluated from the flowfield using Eq. (3) with Eq. (4), which yields

$$P^T(x, t) = \frac{\partial \delta}{\partial x} - \frac{\partial A^T}{\partial x} - \alpha \tag{12a}$$

$$P^B(x, t) = -\frac{\partial \delta}{\partial x} - \frac{\partial A^B}{\partial x} + \alpha \tag{12b}$$

adding these two equations for $x \geq 0$ yields

$$P(x, t) = -\frac{1}{2}\left(\frac{\partial A^T}{\partial x} + \frac{\partial A^B}{\partial x}\right) \tag{13}$$

where

$$A^T(x, t) = u_o(x, t) + \int_0^{\infty} (\tau^T - 1)\,dy \tag{14}$$

and

$$A^B(x, t) = u_o(x, t) + \int_0^{-\infty} (\tau^B + 1)\,dy \tag{15}$$

Because $P^T(x, t) = P^B(x, t)$ for $x \geq 0$, the position of the dividing streamline is obtained by subtracting Eq. (12a) from Eq. (12b) to yield

$$\frac{\partial \delta}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}\{A^T(x, t) - A^B(x, t)\} + \alpha \tag{16}$$

The problem may now be integrated numerically using finite differences and trapezoidal integration. The equations are discretized on a uniform mesh of Δx and Δy. The indices j and k will identify the points in the x and y directions, respectively.

A. Flowfield

The unsteady shear-transport equation given by Eq. (7) is solved using an unconventional Crank-Nicolson method that is partially time implicit. This method allows the shear in the flowfield both above and below the plate and in the wake to be obtained. The time derivative is approximated by the following backward difference:

$$\frac{\partial \tau}{\partial t} = \frac{1}{2(\Delta t)}[(\tau_{j-1,k} + \tau_{j,k}) - (\hat{\tau}_{j-1,k} + \hat{\tau}_{j,k})] \tag{17}$$

The x derivative is approximated by a centered difference

$$u\frac{\partial \tau}{\partial x} = \frac{\bar{u}}{\Delta x}(\tau_{j,k} + \tau_{j-1,k}) \tag{18}$$

which is different than that in Jenson et al.[10]; and the y derivatives are approximated by the following central differences:

$$v\frac{\partial \tau}{\partial y} = \frac{\bar{v}}{4(\Delta y)}[(\tau_{j-1,k+1} + \tau_{j,k+1}) - (\tau_{j-1,k-1} + \tau_{j,k-1})] \tag{19}$$

and

$$\frac{\partial^2 \tau}{\partial y^2} = \frac{1}{2(\Delta y)^2}[(\tau_{j-1,k+1} + \tau_{j,k+1}) - 2(\tau_{j-1,k} + \tau_{j,k}) + (\tau_{j-1,k-1} + \tau_{j,k-1})] \tag{20}$$

The resulting tri-diagonal difference equation is implicit in the y direction and explicit in the x direction. The bar (¯) indicates the velocity at the midpoint between the j and $j - 1$ grid points using the information from the previous time; the caret (^) on the shear terms represents values at the previous time step. Thus, the steady-state solution is obtained by marching in time and in the x direction. The accuracy of the numerical method is of the order $O((\Delta x)^2, (\Delta y)^2, \Delta t)$. The time-marching approach allows the scheme to accommodate reversed flow without the need of a numerical switch as used by Smith,[22] Maozhao,[16] and Elliott and Smith.[21]

The upstream boundary condition used is that of Rizzetta[23]

$$\tau^T = 1 + c_1 \exp(\kappa x) \tag{21a}$$

and

$$\tau^B = -1 + c_2 \exp(\kappa x) \tag{21b}$$

The downstream boundary condition given in Eq. (2b) is enforced by using the asymptotic behavior of the Goldstein wake solution

$$u = x^{1/3} f'(\eta) \text{ for } x \to \infty \tag{22a}$$

where

$$\eta = y/x^{1/3} \tag{22b}$$

B. Centerline Shear

The difference equation for the shear on the dividing streamline is obtained by setting $v = 0$ in the tri-diagonal system for the flowfield. The shear at the first grid point on the dividing streamline is determined by central differencing of Eq. (13) at the trailing edge, which ensures that $P^T = P^B$ at $x = 0$. The integrals are evaluated with trapezoidal integration.

The shear at $y = 0^+$ and $y = 0^-$ is updated at every time step from the wall compatibility condition given by Eq. (11) using central differencing. The upstream boundary condition is given by Eq. (21).

C. Centerline Velocity

The velocity on the dividing streamline is obtained by central differencing of Eq. (10), which yields a tri-diagonal system. The Goldstein wake solution is again used to imply the downstream boundary condition given in Eq. (2b). At $x =$

0^+ the asymptotic behavior given by Daniels[12,24] was implemented as

$$\frac{\partial u_o}{\partial x} = \frac{u_o}{3\Delta x} \tag{23}$$

This condition is used to replace the first derivative term in Eq. (10) for only the first grid point in the wake. The use of this condition ensures proper asymptotic behavior as $x \to 0^+$.

Because of the impulsive nature of the starting solution, the difference equations for the centerline velocity are iterated at each time step, using under relaxation, until the change in u_o is less than 10^{-4}.

D. Dividing Streamline

The position of the dividing streamline is evaluated by first calculating the δ derivative using Eq. (16) and then integrating in x using trapezoidal integration. Note that only $\delta(\Delta x, t)$ is needed for the preceding difference equations; therefore, $\delta(x, t)$ for $x > \Delta x$ is only evaluated at convergence.

E. Computational Procedure

The computation begins by assuming $\tau^{T,B} = \pm 1$ both throughout the flowfield and on $y = 0$ for $x \le 0$. The shear and the position of the dividing streamline is assumed to be zero and $u_o = x^{1/3}$. At each time step, u and v are calculated from the information of the previous time step, which makes the numerical method partly time implicit. The shear flow is then updated above and below the centerline. Next, the position of the dividing streamline $\delta(\Delta x, t)$ is evaluated and is then used to update the shear at $x = \Delta x$. The shear on the plate and on the dividing streamline are computed next, and the centerline velocity is calculated last. This procedure completes one-time iteration and the solution is considered to be converged when $\partial\tau/\partial T < 10^{-4}$ at all grid points.

IV. Results

The present numerical method, as described in Sec. III, was applied to the case of supersonic flow at the trailing edge of a flat plate. Angles of incidence of 0.0 and 0.6790 were first considered in order to check the present scheme by direct comparison with the results of Daniels.[12,24] Figure 2 provides a comparison between the wall shear, pressure, and centerline velocity obtained by the present algorithm for $\Delta x = 0.025$ and $\Delta y = 0.05$ with that of Daniels[24]; the agreement is quite favorable considering Daniels used $\Delta\xi = 0.02$ and $\Delta\eta = 0.1$ in the double region located just aft of the trailing edge, where $\xi = x^{1/3}$ and $\eta = y/x^{1/3}$. For $\alpha = 0.6790$, Daniels[24] gives $\tau(0, 0^+) = 1.0$, $\tau(0, 0^-) = -2.886$, and $P(0) = -0.679$, whereas the present scheme produced $\tau(0, 0^+) = 1.007$, $\tau(0, 0^-) = -2.843$, and $P(0) = -0.685$.

Some studies on the size of Δx and Δy were also performed to assess further the accuracy of the present scheme using $\alpha = 0$. Figure 3 shows the dependency of the wall shear on the size of Δx and Δy, and also the range in Δx where $(\Delta x)^2$ extrapolation could be used. The results indicate that $\Delta x \le = 0.1$ is sufficiently small to permit $(\Delta x)^2$ extrapolation to zero grid size. Figure 4 shows the dependency of centerline velocity near $x = 0^+$ on Δx. As Δx is reduced, the centerline velocity approaches the asymptotic solution given by Daniels.[12] It is clear that a double region at $x = 0^+$ would provide a more accurate solution.

Solutions were obtained for incidence angles beyond stall (i.e., $\alpha > 2.05$) using $\Delta x = 0.05$ and $\Delta y = 0.1$ as the mesh size. Above $\alpha = 2.7$, the scheme became unstable. This instability was first evident in the centerline velocity near $x = 0^+$. The instability could be caused by an actual change in the asymptotic behavior of u_o at $x = 0^+$, which would conflict with the specified behavior. The results for $2.0 \le \alpha \le 2.7$ are presented in Fig. 5 as streamlines spaced at a constant $\Delta(\psi^{1/2})$. The sequence of figures indicate a growth in extent

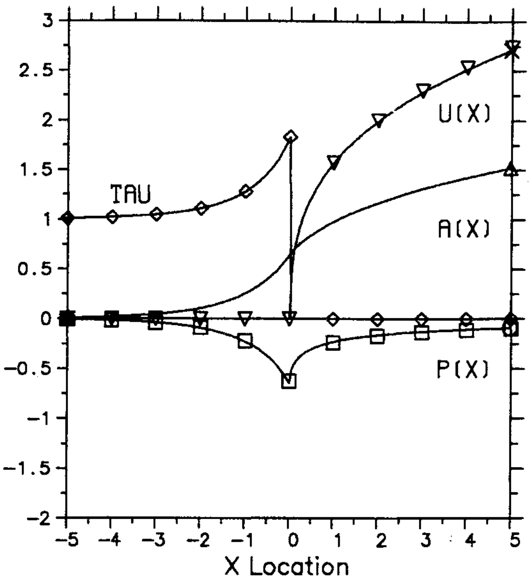


Fig. 2 Comparison of zero angle of incidence results with those of Daniels,[24] which are represented by symbols.

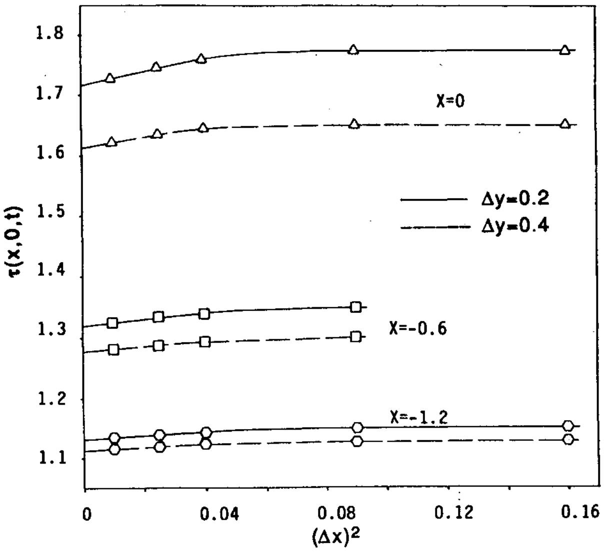


Fig. 3 Order of accuracy check for $(\Delta x)^2$ extrapolation of $\tau(x, 0, t)$ for $\alpha = 0$.

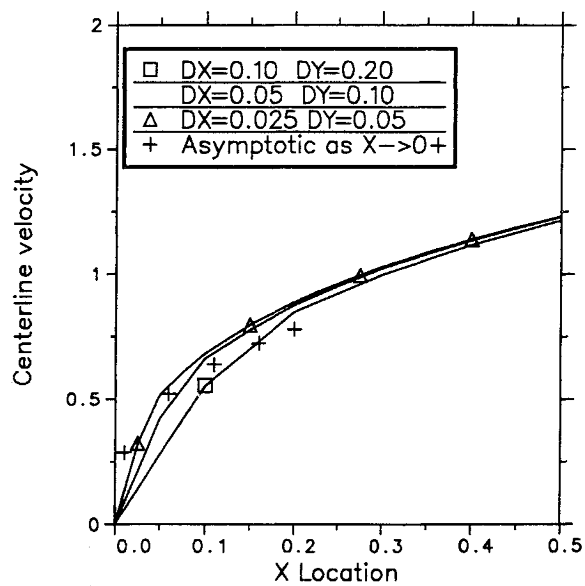


Fig. 4 Effect of grid size on centerline velocity near $x = 0^+$ for zero incidence.

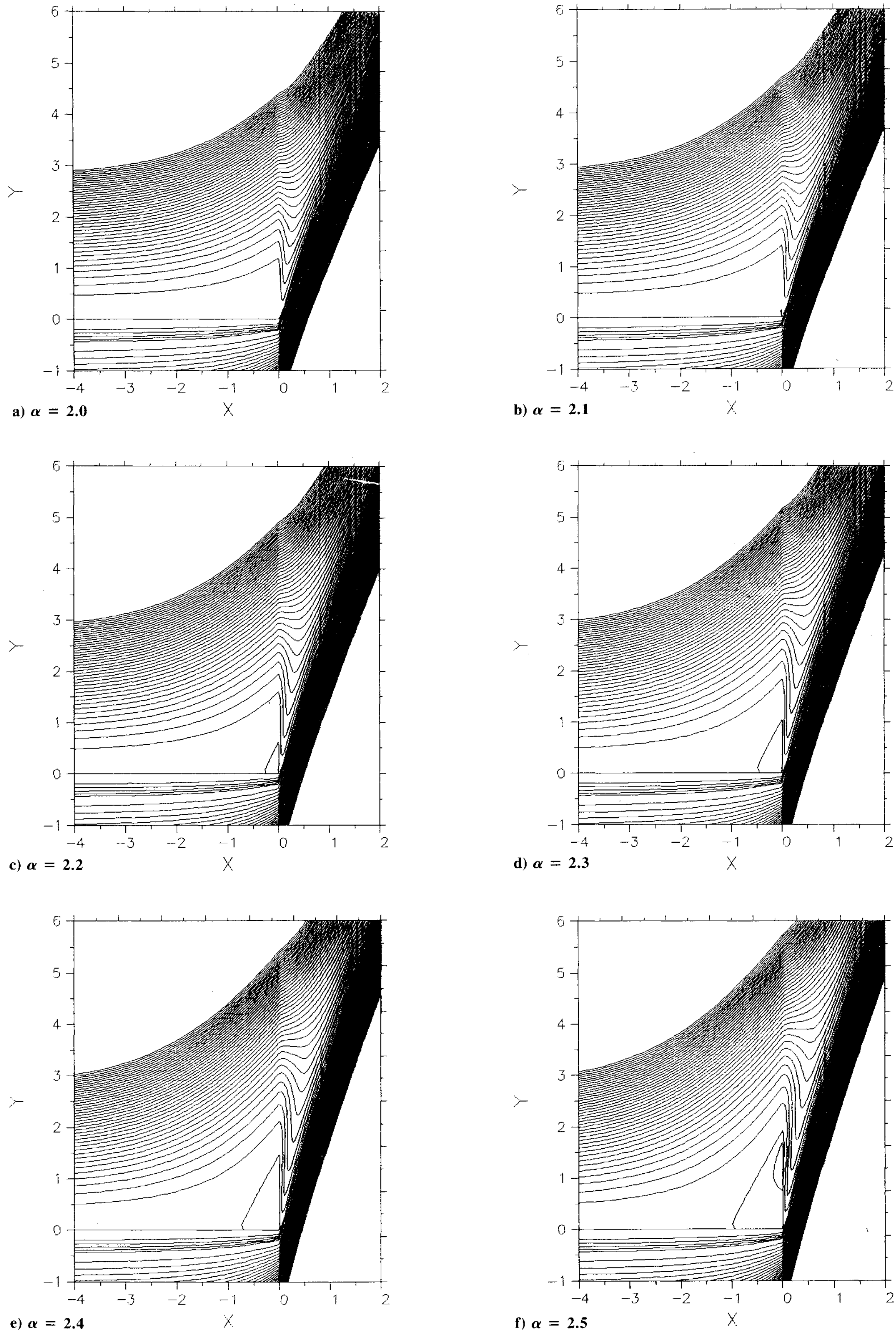
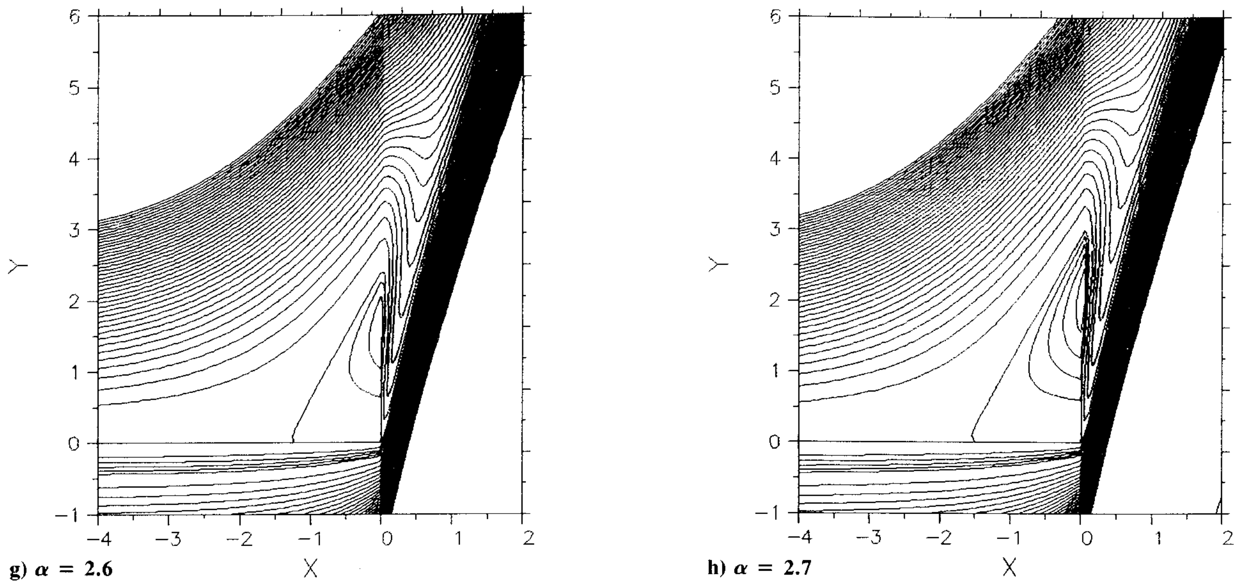


Fig. 5 Streamline results using $\Delta x = 0.05$ and $\Delta y = 0.1$, where $\Delta\Psi^{1/2} = 0.1$ outside the separation region and 0.02 inside the separation region and next to the bottom of the plate.



g) $\alpha = 2.6$ h) $\alpha = 2.7$

Fig. 5 (Continued) Streamline results using $\Delta x = 0.05$ and $\Delta y = 0.1$, where $\Delta\Psi^{1/2} = 0.1$ outside the separation region and 0.02 inside the separation region and next to the bottom of the plate.

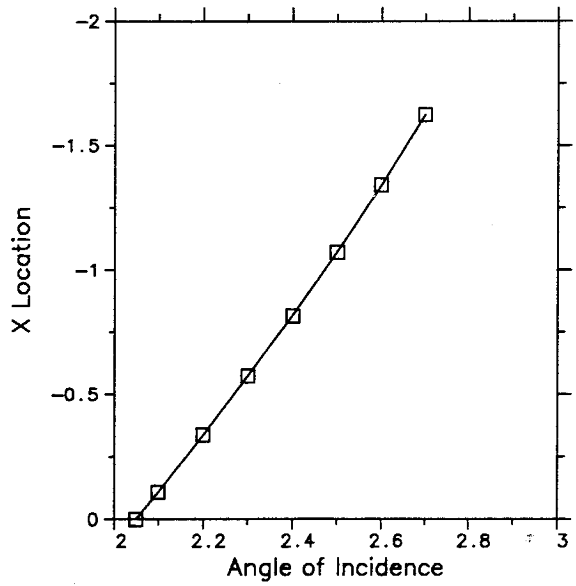


Fig. 6 Separation point as a function of incidence for $\Delta x = 0.05$ and $\Delta y = 0.1$.

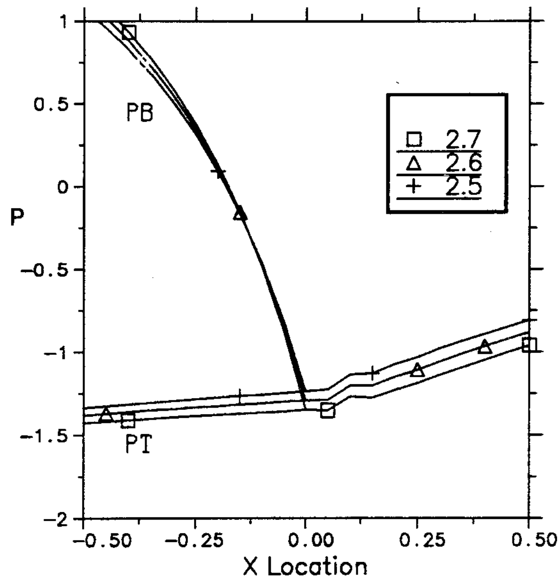


Fig. 8 Pressure results for the upper and lower surfaces near the trailing edge for $\alpha > \alpha_S$.

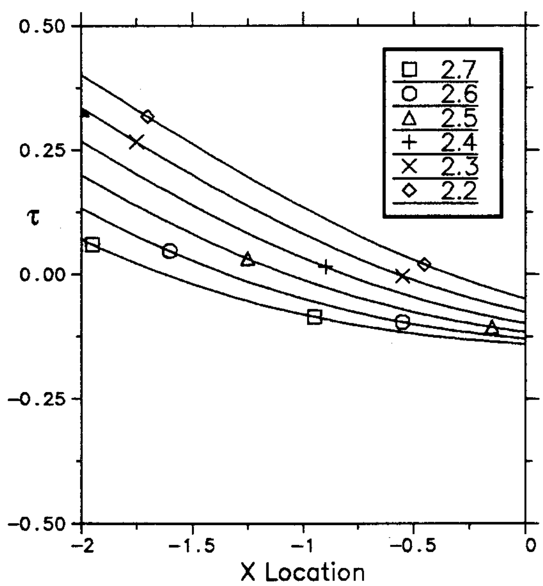


Fig. 7 Wall shear on the upper surface near the trailing edge of the plate for $\alpha > \alpha_S$.

of the separation both upstream and downstream, where the separation point is identified by the intersection of the streamline with the surface of the plate. Similar results have been obtained for subsonic cases by Werle and Verdon,[17] Smith,[22] and Vatsa and Verdon.[18] Some results for the supersonic case were also obtained by Elliott and Smith,[21] but the streamline patterns were indicated only by a single, less detailed figure for the case $\alpha = 2.39$.

Figure 6 shows the separation point location as a function of α. The rate of growth of the length of the separated region increases with α, suggesting a possible approach to catastrophic stall. Alternatively, a possible breakdown of the triple-deck structure at finite α may be anticipated. Smith[25] has proposed a mathematical structure for such a breakdown, involving singularities in the wall shear and the pressure gradient in the reversed-flow region, although the pressure itself and the length of the reversed-flow region remain finite at the breakdown condition. Some evidence in support of this idea is given by the increasingly negative trend of the wall shear occurring in compression-ramp studies,[11] while the present results of wall shear on the upper surface for $\alpha > \alpha_S$ indicate the approach to an asymptotic value at the trailing edge (Fig. 7). The results of Elliott and Smith[21] predict a strongly localized minimum in the wall shear just ahead of the trailing edge with the occurrence of reattachment and positive shear before the trailing edge is reached. The pressure results for the upper and lower surfaces near the trailing edge (Fig. 8) confirm the increase in pressure gradient and finite pressure as the angle of incidence increases. Additional numerical studies are needed to clarify the differences between the present results and those of Elliott and Smith in the behavior of the wall shear at the trailing edge.

V. Conclusions

A time-marching numerical method has been formulated to treat trailing-edge flows based on triple-deck theory. This technique has advantages for solving problems involving separation, for which most steady-flow marching methods (except those that employ switches on the convection term) break down. This work has verified the possibility of flow separation and reattachment at the trailing edge for angles of incidence well beyond stall. The results of this investigation have provided new insight as to the flow structure for trailing-edge separation prior to breakdown. However, additional studies are needed to establish the nature of the breakdown process.

Incorporation of a double region at $x = 0^+$ or a nonuniform mesh and a more exact treatment of the discontinuity at the trailing edge could improve the accuracy and extend the method to higher angles of incidence.

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